Dynamic Thermal Simulation and Modelling of Micro- and Nano-Electronics

Bjorn Vermeersch

Supervisor(s): Gilbert De Mey

Abstract—**As semiconductor devices continuously get both smaller and faster, a profound analysis of the dynamic (time-dependent) thermal behaviour is becoming more and more important. In this article some basic concepts and numerical methods for thermal characterization in the frequency domain are explained. Two practical examples are given. First a simple model is presented for a small package mounted on a thin plate subject to convective cooling. The results are in good agreement with experimental measurements. Next the influence of the finite heat propagation speed is investigated. The calculated temperature distributions give a taste of the thermal wave-like phenomena that are probably to occur in future nano-scaled electronics.**

Keywords— **thermal characterization, dynamic, microelectronics, frequency domain, convective cooling, thermal waves**

I. Introduction

MORE than 40 years after Moore launched his infamous law [1], a still increasing number of transistors is being squeezed onto a single chip. The combination with the high operating speeds leads to enormous heat densities getting dissipated. The heat sources are very small, making electronics a rather peculiar client in the field of heat transfer. Thermal phenomena inside microchips are relatively fast, and in opposition to most other applications (such as heat exchangers, heating of buildings, etc.) thermal conduction rather than convection is the determining and limiting factor. Additionally, semiconductor properties are highly temperature dependent. Hence the heat produced by transistors will influence their electronic characteristics. This so called electrothermal coupling may endanger the reliability of the device, making thermal analysis an important and challenging link in the chain of the chip design process.

II. Dynamic Characterization

In many – especially digital – electronic circuits the electric signals and hence the power dissipation is often periodical. This suggests a description of the dynamic thermal behaviour in the frequency domain. Also other more fundamental reasons can be stated, however we will not go into detail here. For such an approach the temperature response to a hypothetical source dissipating a sinusoidally oscillating (with pulsation ω, rad/s) heat power is investigated. For the mathematical description, phasor notation can be used, similar to the electrical AC analysis well known by electronic engineers. The corresponding differential equation for the temperature distribution T (in K) reads:

$$k\nabla^2 T\left(\vec{r}\right) - j\omega C_v T\left(\vec{r}\right) = -p\left(\vec{r}\right) \tag{1}$$

with k the thermal conductivity (W/mK), C_v the specific heat per volume unit (J/m^{3}K) and p the power density (W/m^3). In some cases the heat equation (1) can be solved analytically, leading to a closed form thermal model. Mostly however numerical or semi-analytical methods are needed. All this will be illustrated by means of two practical examples (sections III and IV).

B. Vermeersch is with the Thermo-electronics Group of the ELIS Department, Ghent University (UGent), Gent, Belgium. E-mail: bjorn.vermeersch@elis.ugent.be .

The thermal diffusion is strongly similar to electrical conduction. The temperature T and heat flow Q (in W) are the counterparts of the voltage and current respectively. By consequence a thermal impedance $Z_{th}(j\omega)$ (in K/W) can be defined, being the temperature of the heat source relative to ambient divided by the dissipated power, both in phasor notation. Z_{th} may be seen as a thermal blueprint of the device. Knowledge of the frequency dependency of the thermal impedance results in a complete yet compact description of the dynamic behaviour.

III. Power Amplifier with Thin Plate Cooling Fin

Experimental setup

A packed chip is screwed to a thin aluminium (k = 200 W/mK, $C_v = 2.4 \times 10^6$ J/m^{3}K) plate, as shown in Fig. 1. The plate acts as a cooling fin: the heat from the chip spreads into the plate by thermal conduction, and can then be transfered to the ambient air. The latter mechanism is called convective cooling. The heat flux q, i.e. the heat flow density (in W/m^2), that can be removed is proportional to the local temperture difference between the plate and the air:

$$q = h\Delta T \tag{2}$$

in which h is the convection coefficient (in W/m^{2}K).

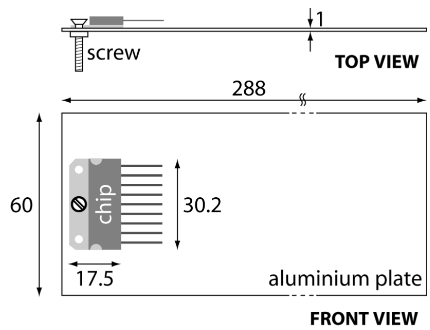

Fig. 1. Power amplifier mounted on a thin plate (all dimensions in mm)

The power amplifier is supplied with a constant current, until a steady state temperature is reached. Then the current is suddenly switched off and from that moment the temperature inside the chip is recorded, by measuring the voltage drop over the protection diode. From this cooling curve the thermal impedance can be obtained using appropriate Fourier techniques [2].

Analytical model

Let us consider a thin plate with thickness t_s, width b and infinite length. At the small side a heat power phasor P is injected. Both the front and rear sides of the plate are subject to convective cooling with convection coefficient h. Using a simple 1-D approximation, the thermal impedance is found as:

$$Z_{th}(j\omega) = \frac{R_0}{\sqrt{1 + j\omega/\omega_0}} \tag{3}$$

where $R_0 = (b\sqrt{2hkt_s})^{-1}$ and $\omega_0 = \frac{2h}{C_v t_s}$.

Results

The thermal impedances obtained from the experiment and the model are compared in Fig. 2. A so called Nyquist representation is used: $\mathrm{Im}\,[\mathrm{Z_{th}(j\omega)}]$ is plotted vs. $\mathrm{Re}\,[\mathrm{Z_{th}(j\omega)}]$ with ω as a parameter. Whereas the model assumed the heat source in direct contact with the plate, here a packed chip is mounted using a screw. Therefore an extra resistance $R_\infty = 0.645\,$K/W, corresponding to the package of the chip, needed to be added to (3). In Fig. 2 a close agreement between theory and experiment is observed. From the model fitting a convection coefficient $h \approx 25\,$W/m^{2}K was derived.

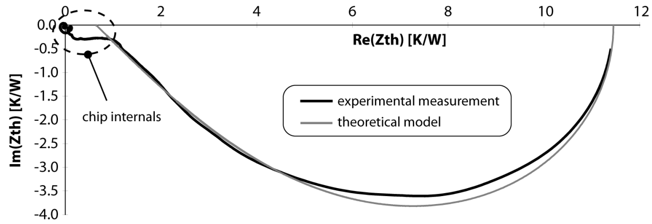

Fig. 2. Thermal impedance of the power amplifier (Nyquist plot)

IV. Non-Fourier Thermal Conduction

Heat conduction is fundamentally a mechanical process: heat spreads through the material as vibrational energy is passed from atom to atom. Hence the speed at which heat can propagate is limited, namely to the speed of sound c in the material. This effect was taken into account by Cattaneo and Vernotte [3]. In (1) an additional factor $1+j\omega\tau$ must be added both to the term with C_v and the power density; τ is a time constant depending on the speed of sound. This gives rise to wave phenomena. The thermal waves are however strongly damped, hence the effect is only noticable in very small devices.

Numerical solution

For the free space and half-infinite substrates, the fundamental solution $G(\vec{r}|\vec{r}')$ of the heat equation is available in closed form. This so called Green's function is the temperature distribution due to a point shaped Dirac impulse source $p = \delta(\vec{r}-\vec{r}')$. For a rectangular source located on top of a substrate, the temperature distribution can then easily be found by superposition:

$$T(\vec{r}) = \iint_{source} p(\vec{r}')G(\vec{r}|\vec{r}')\mathrm{d}\vec{r}' \tag{4}$$

Results

A square heat source measuring 100 nm $\times$ 100 nm with a uniform power dissipation is located on top of a half-infinite silicon substrate ($k = 160\,$W/mK, $C_v = 1.78 \times 10^6$ J/m^{3}K, $c = 2200\,$m/s). The distribution of the T phasor was obtained by evaluating the integral (4). Fig. 3 shows the magnitude of the temperature inside the source for various frequencies.

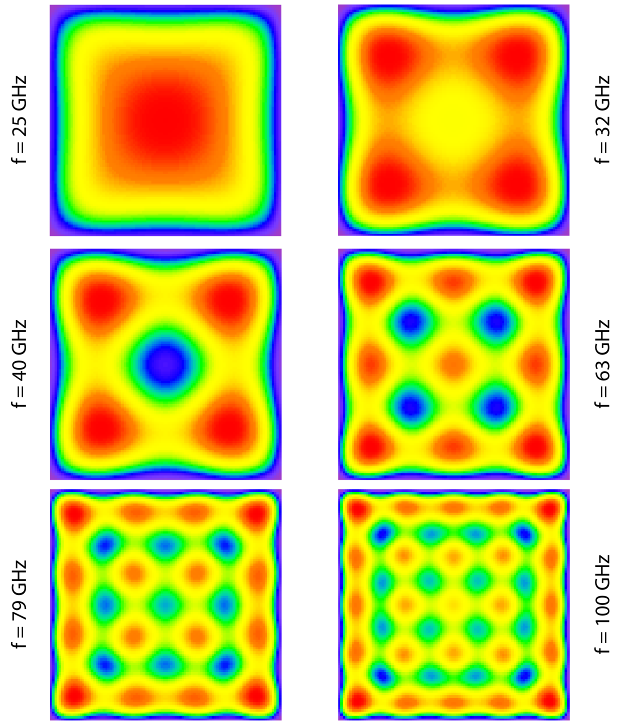

Fig. 3. Calculated $|T|$ distributions inside a 100 nm $\times$ 100 nm heat source (red is 'hottest', purple is 'coldest')

A remarkable observation is the fact that the strongest oscillations (red zones) are not occuring in the center of the source but towards the corners. For very high frequencies the distribution starts to look like an interference pattern.

V. Conclusions

Some general concepts for dynamic thermal characterization were introduced. The presented model for a thin plate under convective cooling conditions showed a good agreement with experimental results. A numerical calculation for a simple structure gave us a glimpse of the thermal wave effects which might be observed in nano-scaled electronic devices.

Acknowledgments

The author wishes to thank the Research Foundation – Flanders (FWO – Vlaanderen) for funding his PhD research.

References

[1] G. E. Moore, *Cramming more components onto integrated circuits*, Electronics 38(8), 1965.
[2] P. Kawka, *Thermal impedance measurements and dynamic modeling of electronic packages*, PhD thesis, Ghent University, 2005.
[3] L. Codecasa, *Thermal networks from heat wave equation*, IEEE Trans. Comp. Pack. Technol. 28(1): pp. 14 - 22, 2005.